NASA TECHNICAL NOTE

NASA TN D-6516

MEASUREMENTS OF $\eta\rho_n$ IN AN INTERVAL ABOUT T_λ FOR PURE HELIUM-4 AND FOR THREE DILUTE MIXTURES OF HELIUM-3 IN HELIUM-4

by Rayjor W. H. Webeler and Gabriel Allen

Lewis Research Center
Cleveland, Ohio 44135

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • SEPTEMBER 1971

1. Report No. NASA TN D-6516	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle MEASUREMENTS OF $\eta\rho_n$ IN AN INTERVAL ABOUT T_λ FOR PURE HELIUM-4 AND FOR THREE DILUTE MIXTURES OF HELIUM-3 IN HELIUM-4		5. Report Date September 1971 6. Performing Organization Code
7. Author(s) Rayjor W. H. Webeler and Gabriel Allen		8. Performing Organization Report No. E-6448
9. Performing Organization Name and Address Lewis Research Center National Aeronautics and Space Administration Cleveland, Ohio 44135		10. Work Unit No. 112-02 11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Technical Note 14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

Measurements of the product of viscosity η and normal density ρ_n have been made for pure He^4 and for three He^3-He^4 mixtures (He^3 mole fractions x_3 of 0.005, 0.0475, and 0.1005) over temperature intervals which included their respective lambda points. The measurements were made using a torsional crystal technique in which a piezoelectric, cylindrical quartz crystal was immersed in the sample fluid and driven at its resonant frequency in a fundamental torsional mode of vibration. The precision of the $\eta\rho_n$-against-T measurements for the mixture was greater than any previous measurements in the vicinity of T_λ. Doubly reduced plots of $\eta\rho_n$ against T showed a slight but definite tendency for such curves to rotate clockwise about the (1, 1) point with increasing x_3 up to about 0.05. Values of η were derived from the measurements by computing ρ_n as a function of reduced temperature. For each mixture, both η and $d\eta/dT$ were continuous across T_λ. No tendency was manifest for such curves to develop discontinuities as x_3 decreased. It was, therefore, inferred that both η and $d\eta/dT$ of pure He^4 are continuous across $T_\lambda(0)$. A further result obtained here is that up to mole fractions of 0.1, the lambda viscosity of the mixture $\eta_\lambda(x_3)$ bears a linear relation to the lambda temperature $T_\lambda(x_3)$.

17. Key Words (Suggested by Author(s)) Viscosity — Dilute solutions of helium 3 Low temperature — Lambda point He^4	18. Distribution Statement Unclassified - unlimited

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 20	22. Price* $3.00

* For sale by the National Technical Information Service, Springfield, Virginia 22151

MEASUREMENTS OF $\eta\rho_n$ IN AN INTERVAL ABOUT T_λ FOR PURE HELIUM-4 AND FOR THREE DILUTE MIXTURES OF HELIUM-3 IN HELIUM-4

by Rayjor W. H. Webeler and Gabriel Allen

Lewis Research Center

SUMMARY

Measurements of the product of viscosity η and normal density ρ_n have been made for pure He^4 and for three He^3-He^4 mixtures over temperature intervals which included their respective lambda points. The mole fractions of He^3 in the mixtures were 0.005, 0.0475, and 0.1005. The measurements were made using a torsional crystal technique in which a piezoelectric, cylindrical quartz crystal was immersed in the sample fluid and driven at its resonant frequency in a fundamental torsional mode of vibration.

A plot of $\eta\rho_n$ against T was made for all the mixtures and for pure He^4. The general shape of the curves was independent of He^3 concentration. However, near T_λ, the value of $\eta\rho_n$ decreased monotonically with increasing x_3 at a given value of $T - T_\lambda$, where T_λ is the lambda point of the mixture.

Doubly reduced plots of $\eta\rho_n$ against T showed that there is a slight but definite tendency for the curve to rotate clockwise about the (1, 1) point on such a plot, with increasing x_3 up to about 0.05.

Values of ρ_n were computed as a function of reduced temperature T^* and used to obtain curves of the viscosity as a function of T^*. For both the 0.5 percent and 10 percent mixtures, the η-against-T^* curve was precise enough to conclude that η and $d\eta/dT$ are continuous across the relevant $T_\lambda(x_3)$, considering a precision of 0.1 millikelvin in the temperature determination. Such precision was absent for the pure He^4. However, at larger values of $|T - T_\lambda|$, where the pure He^4 measurements are reasonably precise, the results blended in well with the results for the mixtures. Furthermore, the η-against-T^* curves for the mixtures showed no tendency to develop discontinuities in η or $d\eta/dT$ as x_3 decreased. For these reasons, it has been inferred that both η and $d\eta/dT$ are continuous across $T_\lambda(0)$ for pure He^4.

A further result reported herein is that, up to concentrations of 10-percent He^3, the viscosity at the lambda point of the mixture $\eta_\lambda(x_3)$ bears a linear relation to the lambda temperature of the mixture $T_\lambda(x_3)$. Since $T_\lambda(x_3)$ is given by $x_4^{2/3}T_\lambda(0)$ to a good approximation, this result is tantamount to asserting that $\eta_\lambda(x_3) - \eta_\lambda(0)$ is proportional to $\left(1 - x_4^{2/3}\right)T_\lambda(0)$.

INTRODUCTION

Although measurements of the viscosity of liquid He^4 have been carried out for more than 60 years over a wide range of experimental conditions, good reasons for further investigations still remain. First of all, there is a wide divergence among published viscosity values η below 1.3 K (ref. 1). Secondly, while there is much better agreement near 2 K, the question concerning the continuity of η and/or $d\eta/dT$ across T_λ remains unanswered, and the existing data are rather sparse near T_λ.

One experimental problem in obtaining viscosity data in the neighborhood of T_λ is the determination of the temperature of the sample fluid. For $T < T_\lambda$, the very high heat-transport properties of liquid He^4 makes it an ideal heat-reservoir material. But very close to T_λ its heat-transport properties rapidly decrease, and about T_λ they are poor. If the sample fluid and bath reservoir are both He^4, they will both pass through their lambda points at about the same time. This will destroy the isothermal environment of the sample fluid, making it difficult to obtain a precise unambiguous temperature.

To overcome this difficulty, the lambda point of the sample fluid can be depressed below that of the reservoir. Then the reservoir maintains excellent heat-transport properties on both sides of the lambda point of the sample fluid. This can be accomplished by keeping the sample fluid at a higher pressure than the bath, but this method can result in other difficulties (ref. 2). Another way to depress the lambda point is to add some He^3 to the sample fluid. This method, which was used in the present investigation, enables highly reproducible measurements to be made on the sample fluid in the immediate vicinity of and across its lambda point. However, the data are then measurements of the He^3-He^4 mixtures, rather than of pure He^4.

This last point is not a disadvantage, of course. In the first place, such data on the mixtures are of interest in themselves. The only previously published viscosity values of He^3-He^4 mixtures in temperature intervals which include their respective lambda points are the rather sparse data of Dash and Taylor (ref. 3). Staas et al. (ref. 4) reported viscosity values as a function of temperature for He^3-He^4 mixtures over a wide range of concentrations of He^3. But all these latter measurements, are below the lambda points of the respective mixtures. Their absolute values also appear to be about 20 percent too low. A fairly comprehensive list of references to previous work can be found in reference 5. In the second place, the present measurements turned out to be so precise that some inferences about pure He^4 could be obtained by extrapolation from the dilute mixtures. These inferences, which concern the continuity of η and $d\eta/dT$ across $T_\lambda(0)$, could not be made from the available data for pure He^4.

In this investigation, using the torsional crystal technique, measurements were made of $\eta\rho_n$ as a function of temperature for pure He^4 and for He^3-He^4 mixtures con-

taining three different concentrations of He^3: x_3 of 0.005, 0.0475, and 0.1005. Here, ρ_n is the normal density of the mixture.

After examining some alternatives, it was decided that a very useful way to compare results for different concentrations of He^3 was to make doubly reduced plots of the data. In these plots, not only the temperature but also the ordinates are expressed as multiples of the lambda point values. On such a plot, the lambda transition is always located at (1, 1). The use of such plots for the mixtures and pure He^4 in the present investigation enabled us to determine the (1, 1) point more accurately than had been done in reference 6. In fact, we were able to determine a temperature correction for data given in reference 6.

Furthermore, the availability of accurate values of the normal density of pure He^4 enabled the viscosity to be derived from our measured values of $\eta\rho_n$. An examination of the η-against-T* plots established the inference that there is no discontinuity in either η or $d\eta/dT$ across $T_\lambda(0)$ for pure He^4,

SYMBOLS

M_3	molar mass of He^3, kg/mole
T	temperature, K
T^*	reduced temperature of the mixture, $T^* = T/T_\lambda$
$T_\lambda(0)$	experimental lambda temperature of pure He^4
V_3	partial molar volume of He^3 in the mixture, m^3/mole
V_4	partial molar volume of He^4 in the mixture, m^3/mole (Here, V_4 is taken to be equal to the molar volume of pure He^4 at $T_\lambda(0)$.)
V_λ	molar volume of the mixture at its lambda point, $x_3V_3 + x_4V_4$
x_3	mole fraction of He^3
x_4	mole fraction of He^4
γ	number obtained by fitting a straight line to an experimental plot of an effective hydrodynamic density against the density of a superfluid, $\gamma = 0.796$
η	viscosity, N-sec/m^2
ρ_3	M_3/V_3
$\rho_n(T^*, 0)$	normal density of pure liquid He^4 at a value of $T^*(0)$ equal to T^* of He^3-He^4 mixture (denoted by 3-4); experimental temperature corresponding to T^* in term $\rho_n(T^*, 0)$ is $T_\lambda(0)T^* \neq T_\lambda(x_3)T^*$

ρ_λ^0 density of pure He^4 at $T_\lambda(0)$, kg/m^3

φ_3 volume fraction of He^3 in the mixture, x_3V_3/V_λ

φ_4 volume fraction of He^4 in the mixture, $1 - \varphi_3$

Superscript:

* reduced quantity

EXPERIMENTAL PROCEDURE AND APPARATUS

A piezoelectric, cylindrical quartz crystal was immersed in the liquid of interest and driven in a torsional mode of vibration at its resonant frequent of 11 kilohertz. The logarithmic decrements were then determined as a function of temperature by measuring the crystal resistance at resonance. This method of measuring the viscosity was first used by Welber and Quimby (ref. 7). As a result of a number of refinements in the apparatus, the sensitivity and precision of the method have been greatly improved (ref. 5).

A silver crystal holder served as a support for four pure silver electrode quadrants that surround, but do not contact, the crystal (fig. 1). The crystal was 0.53 centimeter in diameter by 17.7 centimeters long and was supported by two nylon threads at its central displacement node. The space between the electrode quadrants and the crystal surface (about 0.05 cm) was filled by the mixture of interest, and it was this portion of the total amount of fluid (about 0.75 mole) in the copper sample chamber which provided the measured decrement.

The crystal resistance was measured by capacitance coupling to the electrode quadrants, which thereby eliminated the need for a mechanical contact. These quadrants were connected to one arm of a modified Schering capacitance bridge by small electrical wires running inside a capillary tube from the sample chamber to room temperature. A PAR two-phase lock-in amplifier was used to detect the null bridge balance for small driving voltages (4 to 9 mV) across the crystal at the crystal resonant frequency.

The low-temperature apparatus (fig. 2) used in this investigation was originally designed for measurements near 0.1 K rather than specifically for measurements near 2 K. However, some minor modifications made it suitable for the lambda point measurements for which it was used. By drilling a number of holes in the bottom of the brass chamber of the apparatus, the middle chamber (which surrounds the sample chamber) filled with liquid He^4 from the bath reservoir. This He^4 in the middle chamber was in contact with the copper sample chamber and served as a heat exchanger between the reservoir and the sample chamber. In this way, the temperature of the sample chamber was regulated by the temperature of the liquid He^4 in the reservoir. The tem-

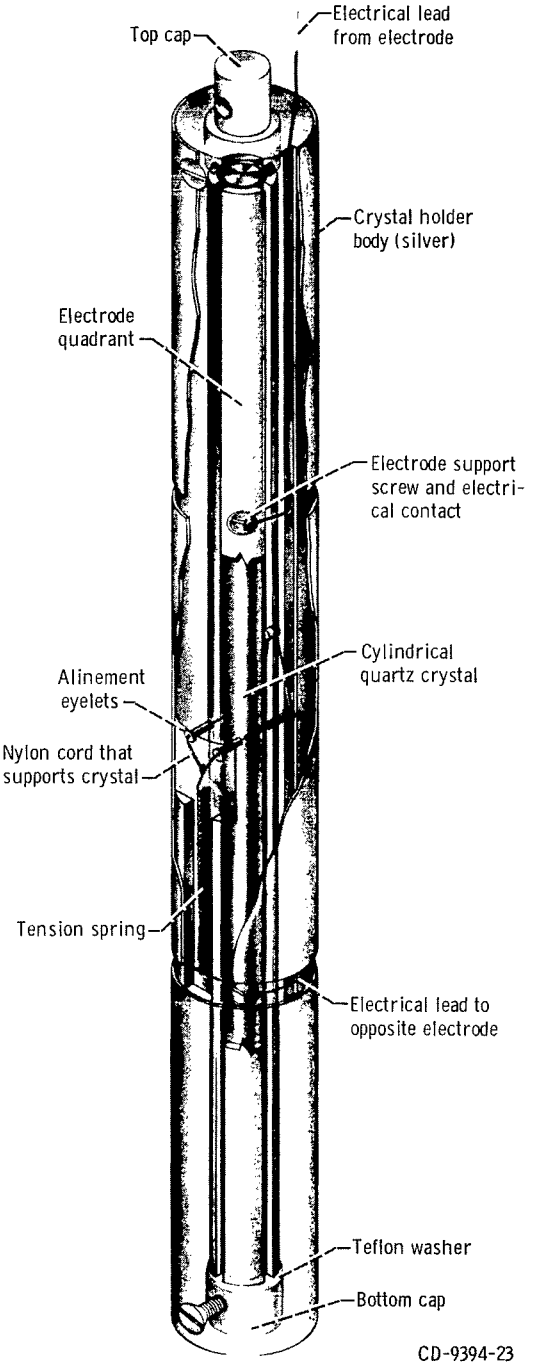


Figure 1. - Torsional crystal suspension system prior to placement in sample chamber of figure 2.

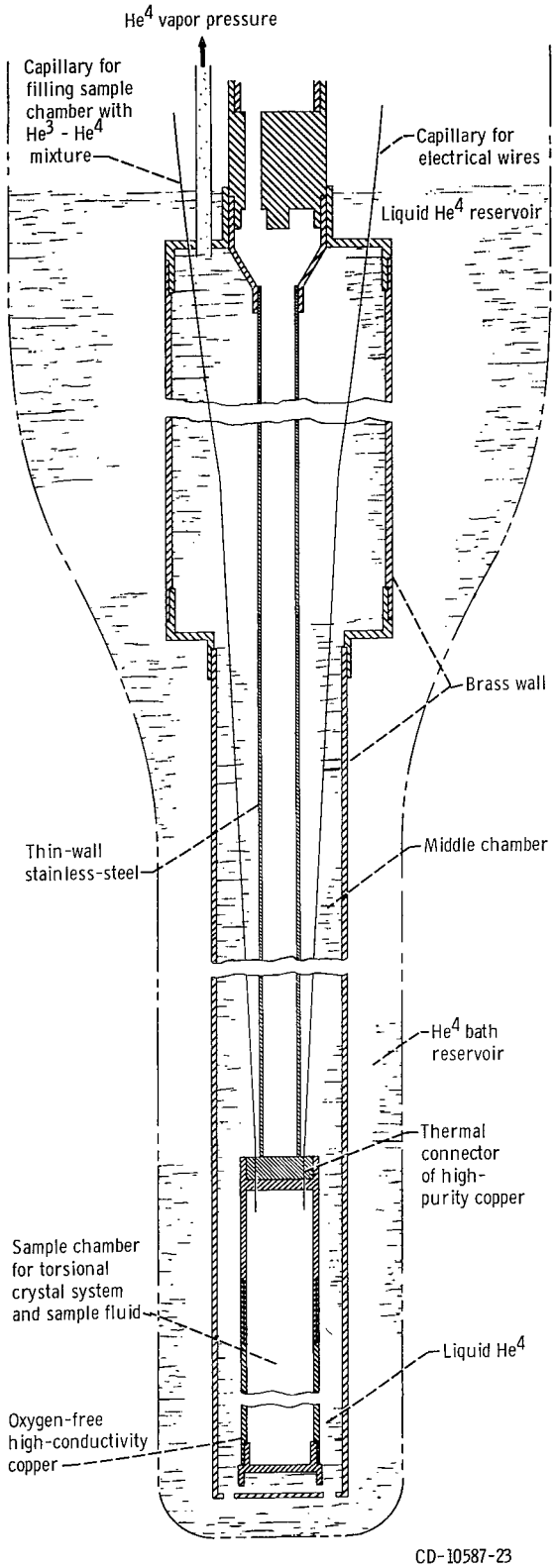


Figure 2. - Schematic of low-temperature apparatus.

perature of the bath reservoir was controlled by a Texas Instruments Bourdon Tube Pressure Gauge (TIBTPG), a Texas Instruments Pressure Controller (TIPC), two pumping systems for the reservoir, and an auxiliary He^4 gas supply. The latter was bled into the reservoir system at a rate determined by the TIPC. The TIBTPG sensed the vapor pressure of the liquid He^4 in the middle chamber with a sensitivity better than one part in 10 000. Temperature changes less than 0.1 millikelvin could be detected.

The temperature controller functioned as follows: The TIBTPG was set to provide a zero error signal at a given vapor pressure corresponding to the temperature at which data were to be taken. If the vapor pressure in the middle chamber deviated from the chosen setting, the TIBTPG produced an error signal which was sensed by the servomechanism in the TIPC and accordingly adjusted the flow of He^4 gas into the reservoir system and pumping lines. This method of sensing the vapor pressure of the middle chamber and using the flow of He^4 gas into the reservoir system to change the temperature of the bath will not work above the lambda point temperature of the reservoir. On the other hand, it works very well near but below the lambda transition.

It was necessary to have a primary pumping system in addition to the pumping system required in conjunction with the TIPC. When both pumping systems and the temperature control system were operating properly, no temperature oscillations were detectable. The electrical resistance of the crystal varied rapidly with temperature just below $T_\lambda(x_3)$ due to the rapidly changing values of $\eta\rho_n$. In fact, for dilute mixtures of He^3 in He^4, a small change in its temperature near but below $T_\lambda(x_3)$ was accompanied by an-order-of-magnitude-greater change in $\eta\rho_n$. The crystal resistance could be measured to a few parts in 30 000, so that such measurements provided another more sensitive means of checking on the temperature stability of the temperature control system to supplement the measurement of just the temperature or the vapor pressure.

The sample temperature was determined from both resistance thermometer measurements and vapor pressure measurements of the liquid He^4 in the middle chamber. The useful thermometers in the sample chamber consisted of four nominal 10-ohm, 0.1-watt Allen Bradley carbon composition resistors. These thermometers were mounted in the sample chamber in a previous experiment and so were retained even though their dR/dT was not optimum near 2 K. They were wired electrically in parallel and calibrated by means of vapor pressure measurements from liquid He^4 in the middle chamber. A fused quartz Bourdon tube with a range of 0.0 to 3.37×10^4 newtons per square meter (0.0 to 253 mm Hg) was in the TIBTPG which was used to measure the vapor pressure. The absolute temperature could only be determined to within 2 millikelvin, but changes less than 0.1 millikelvin were detectable.

If the middle chamber contained liquid He II, the TIBTPG determined the temperature of the sample chamber with better precision than the nominal 10-ohm Allen Bradley resistance thermometers. (Temperature changes could be determined with a

precision of ±0.1 mK using the resistance thermocouples.) However, this was not true when the middle chamber contained He I. At and above the lambda point of the He^4 in the middle chamber and reservoir, only the resistance thermometers were useful in determining sample chamber temperature.

Since the absolute temperature of the sample could not be determined more accurately than to 1 to 2 millikelvins, the measured temperature could not be used to locate the lambda transition for pure He^4 or any of the mixtures with sufficient accuracy. The experimental lambda point for any particular mixture was chosen by carefully locating the temperature at which a discontinuity in the slope of the $\eta\rho_n$ occurred. This method of locating the lambda point was not used for pure He^4 due to the large scatter of the pure He^4 data. For pure He^4, the lambda point was located by adding a correction to the very precisely determined experimental lambda temperature for the 0.5-percent mixture. The experimental $T_\lambda(0)$ used was $T_\lambda(0.005) + |\Delta T|$, where ΔT is the amount by which the lambda point of pure He^4 differs from that of the 0.005-mole-fraction mixture. This decrease follows the 2/3 power of x_4 (ref. 3) with sufficient accuracy.

The composition of the mixtures was determined by first introducing a measured pressure of He^3 gas into an evacuated tank, and then introducing pure He^4 gas from a high-pressure source until the desired composition resulted (assuming ideal-gas behavior). The accuracy with which the composition was prepared was limited only by the accuracy with which the pressures could be determined from the TIPC. This was better than 2 percent for the 0.005-mole-fraction sample which was the least accurate. However, this, in general, was not necessarily a true indication of the accuracy of the composition in the sample chamber during the measurements. The sample volume was comparatively large (3/4 mole of mixture), and the liquid nearly filled the entire sample chamber except for a capillary. This procedure kept the error due to vaporization very small.

After the data were taken, the composition of the 0.005- and 0.1005-mole-fraction mixtures was checked to ensure that no leaks to the He^4 reservoir had occurred during the course of the measurements. The contents of the sample chamber were collected during warmup to room temperature, and a random sample was analyzed with a mass spectrometer designed to do simple gas analysis. The resolution of the spectrometer was ±0.0003 for the 0.005-mole-fraction mixture and ±0.002 for the 0.1005-mole-fraction mixture.

Due to the presence of a capillary plug (which was noticed on filling the sample chamber), the composition of the 0.0475-mole-fraction mixture had to be determined in an entirely different way. It was determined by comparing the absolute values of $\eta\rho_n$ with values determined in a previous experiment on a sample of slightly lower but known composition. We estimate the uncertainty in composition of this mixture to be ±0.001 mole fraction.

DETERMINATION OF THE NORMAL DENSITY OF THE MIXTURES

The physical quantity measured in the usual "viscosity" measurements on liquid helium is the product of viscosity and the normal density of the fluid $\eta\rho_n$. In order to know η alone, it is necessary to find ρ_n. From the results of the difficult measurements of ρ_n, the necessary ingredients for the computation were set forth in a classic paper by Dash and Taylor (ref. 3). In this paper, it was shown that the normal density of a He^3-He^4 mixture could be expressed as

$$\rho_n(T^*, x_3) = \varphi_3\rho_3 + \frac{\varphi_4}{1 + \gamma\varphi_3}\left[\rho_n(T^*, 0) + \gamma\varphi_3\rho_\lambda^0\right] \tag{1}$$

where

T^*	reduced temperature of the mixture, $T^* = T/T_\lambda$
φ_3	volume fraction of He^3 in the mixture, $\varphi_3 = x_3(V_3/V_\lambda)$
φ_4	$1 - \varphi_3$
M_3	molar mass of He^3, kg/mole
V_λ	$x_3V_3 + x_4V_4$
V_3	partial molar volume of He^3 in the mixture
V_4	partial molar volume of He^4 in the mixture
ρ_3	M_3/V_3
γ	number obtained by fitting a straight line to an experimental plot of effective hydrodynamic density against density of superfluid, $\gamma = 0.796$
ρ_λ^0	density of pure He^4 at $T_\lambda(0)$
$T_\lambda(0)$	experimental lambda point of pure He^4
$\rho_n(T^*, 0)$	normal density of liquid He^4 at value of $T^*(0)$ equal to $T^*(x_3)$ of the He^3-He^4 mixture (denoted by 3-4); experimental temperature corresponding to T^* in term $\rho_n(T^*, 0)$ is $T_\lambda(0)T^* \neq T_\lambda(x_3)T^*$

The reasoning on which equation (1) was based still appears valid. However, there now exist improved measurements of some of the quantities needed to compute some of the variables on the right side of the equation. To begin with, in reference 3, the value of V_3 was given as 35.552×10^{-3} cubic meters. This value was based on a value of 38.456×10^{-3} cubic meters for the molar volume of pure He^3 at 2 K, a convenient temperature near $T_\lambda(x_3)$ of the mixtures. Subsequently, Kerr and Taylor (ref. 8) made a

more precise measurement of this quantity and obtained a value of 38.854×10^{-3} cubic meters. Accordingly, the value of V_3 was proportionately adjusted to 35.920×10^{-3} cubic meters in computing φ_3 and V_3. Using this adjusted value of V_3 and the value of 3.01603 for the atomic weight of He^3 on the C^{12} scale, a value of 83.97 kilograms per cubic meter (0.08397 g/cm^3) is obtained for ρ_3.

Using the same apparatus in which the improved measurements on He^3 were made, Kerr and Taylor (ref. 9) also made improved measurements of the molar volume of He^4. The value obtained was 27.387×10^{-3} cubic meters and is used here. This completes the modifications to the values given in reference 3 which were used in calculating the first term on the right side of equation (1).

Some modifications to the values given in reference 3 were made in the second term on the right side of equation (1) also. Clow and Reppy (ref. 10) showed that, for $T_\lambda - T \leq 0.060$ K,

$$\frac{\rho_s}{\rho_4} = 1.438\left[T_\lambda(0) - T\right]^{2/3}$$

where ρ_4 is the density of pure liquid He^4 and ρ_s is the density of the superfluid component. Very soon afterwards, Tyson and Douglass (ref. 11) made a more precise measurement of the exponent and determined its value as 0.666 ± 0.006. Therefore the following expression was used to compute $\rho_n(T^*, 0)$:

$$\rho_n(T^*, 0) = \rho_4(T^*)\left\{1 - 1.438\left[T_\lambda(0) - T\right]^{0.666}\right\} \qquad \text{for } T^* < 1$$

$$\rho_n(T^*, 0) = \rho_4(T^*) \qquad \text{for } T^* \geq 1$$

Here, $\rho_4(T^*)$, the total density of liquid He^4 at T^*, has a maximum at about $T^* = 1.00276$ and changes very little for $T^* > 1.00276$. The expansion coefficient in the vicinity of $T_\lambda(0)$ is about 0.01. Therefore, $\rho_4(T^*)$ was computed from the relation:

$$\rho_4(T^*) = \rho_\lambda^0\left[1 - 0.01(1.00276 - T^*)\right] \qquad \text{for } T^* < 1.00276$$

$$\rho_4(T^*) = \rho_\lambda^0 \qquad \text{for } T^* \geq 1.00276$$

where ρ_λ^0 is the density of pure He^4 at $T^* = 1$ and has a value of 146.207 kilograms per cubic meter. For dilute solutions of He^3, the term involving $\rho_n(T^*, 0)$ is the largest of the three terms on the right side of equation (1). Thus, the papers of Clow and Reppy

(ref. 10) and Tyson and Douglass (ref. 11), which established the variation of ρ_s with the 2/3 power of $T_\lambda(0) - T$, were of considerable importance in that $\rho_n(T^*, 0)$ could now be computed accurately.

To summarize, the following relations were used to compute the normal component of the 3-4 density:

$$\rho_n(T^*, x_3) = \varphi_3\rho_3 + \frac{\varphi_4}{1 + \gamma\varphi_3}\left[\rho_n(T^*, 0) + \gamma\varphi_3\rho_\lambda^0\right]$$

$$\varphi_3 = \frac{x_3}{x_3 + 0.76244(1 - x_3)}$$

$$\varphi_4 = 1 - \varphi_3$$

$$\gamma = 0.796$$

$$\rho_3(T^*) = 83.97$$

$$\rho_n(T^*, 0) = 146.207\left[1 - 0.01(1.00276 - T^*)\right]\left[1 - 2.41175(1 - T^*)^{0.666}\right] \quad \text{for } T^* < 1$$

$$\rho_n(T^*, 0) = 146.207\left[1 - 0.01(1.00276 - T^*\right] \quad \text{for } 1 \le T^* < 1.00276$$

$$\rho_n(T^*, 0) = 146.207 \quad \text{for } T^* \ge 1.00276$$

The normal component of pure He^4 is obtained by using $\rho_n(T^*, 0)$. A graph of $\rho_n(T^*, \varphi_3)$ computed from this formula is shown in figure 3 for the 0.5-percent solution.

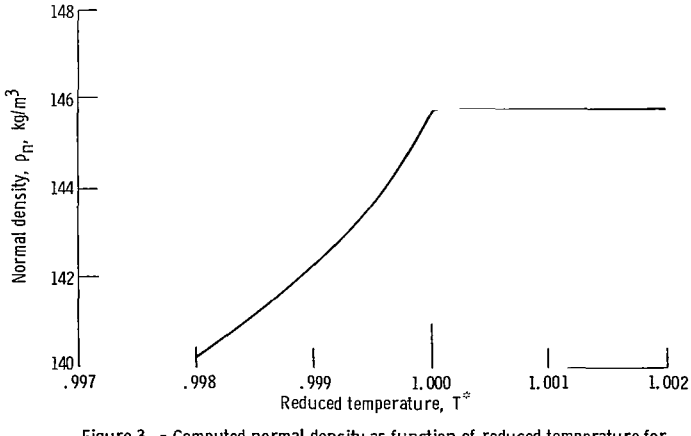


Figure 3. - Computed normal density as function of reduced temperature for 0.5-percent-He^3 mixture.

RESULTS AND DISCUSSION

The measured values of $\eta\rho_n$ against T_{58} for the three He^3-He^4 mixtures and for pure He^4 are shown in figure 4. The results show clearly that $\eta\rho_n$ is a monotonic decreasing function of He^3 concentration. Although this behavior is to be expected qualitatively since the addition of He^3 decreases the density of the mixture, the viscosity itself decreases monotonically with increasing He^3 concentration at any fixed value of $T - T_\lambda(x_3)$ within the range of temperature and composition considered in this investigation (see fig. 6).

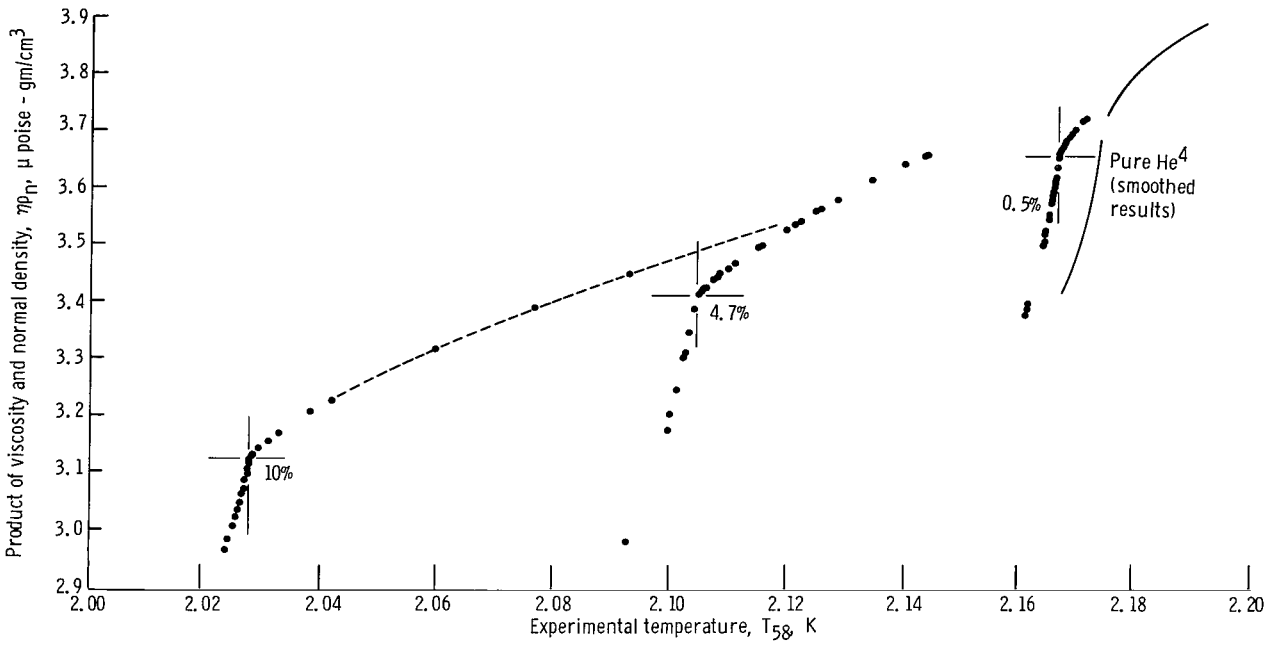
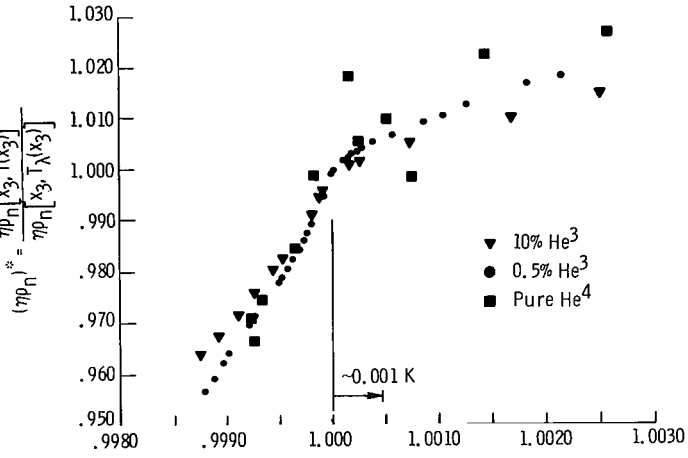


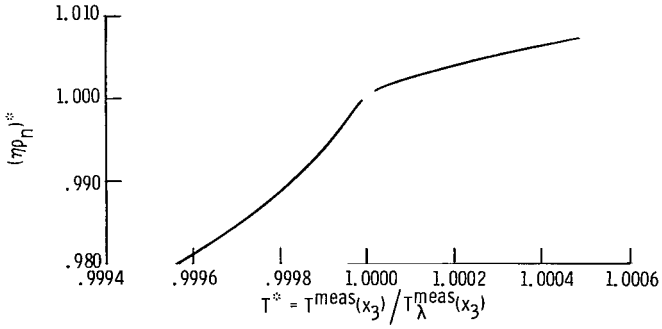
Figure 4. - Product of viscosity and normal density as function of experimental temperature for pure He^4 and dilute He^3-He^4 mixtures. (Precision in T_{58} is better than ±0.1 millikelvin; the absolute T_{58} is uncertain within ±2 millikelvins; precision in $\eta\rho_n$ is about 0.05 percent. The approximate T_{58} absolute temperature may be obtained by subtracting 1.7 millikelvins from experimentally determined temperatures shown on abscissa.)

The functional dependence on T of all the $\eta\rho_n$ curves seems to be qualitatively similar, so some quantitative measure of the similarity among different concentrations would be of interest. As was indicated in the INTRODUCTION, doubly reduced plots provide such a measure. A doubly reduced plot is one on which both the ordinate and the abscissa are expressed in multiples of the respective values of the unreduced quantity at $T_\lambda(x_3)$. On such a plot, the transition between the normal and the superfluid state is at the location (1, 1) for all concentrations. A superscript * is used to distinguish a reduced quantity from the corresponding unreduced quantity.

A plot of $(\eta\rho_n)^*$ against T^* is shown in figure 5(a) for pure He^4, for a mixture containing 0.5-percent He^3 and for another mixture containing 10-percent He^3. Data for the 4.7-percent mixture are not included because they fall very close to the 10-percent data and would make it more difficult to discern the change in the curves as a function of concentration. On this plot, the curves are indeed very similar for all the mixtures. However, there is a clear tendency for the curves to rotate clockwise about the (1, 1) point with increasing x_3.



(a) Wide range.



(b) Expanded and smoothed plot for 0.5-percent-He^3 mixture.

Figure 5. - Reduced product of viscosity and normal density as function of reduced temperature.

An expanded and smoothed $(\eta\rho_n)^*$-against-T^* plot is shown in figure 5(b) for the 0.5-percent mixture. The experimental lambda point for the 0.5-percent curve could be determined to within ±0.1 millikelvin. (Of course, this is not the accuracy.) The absolute temperature of the measurements is known only to within 2 millikelvin. Simi-

larly, the precision of the values of $(\eta\rho_n)$ is approximately 0.05 percent, whereas the accuracy was not determined.

The precision of the results for the mixtures is clearly very much better than for pure He^4 (see fig. 5(a)). In particular, it was possible to determine $T_\lambda(x_3)$ from these curves with reasonable confidence when x_3 was nonzero. Therefore, the values of $T_\lambda(x_3)$ which were used to analyze the data and to determine $T^*(x_3)$ were obtained from these curves. This procedure was clearly unsatisfactory for pure He^4, so that $T_\lambda(0)$ was obtained by extrapolating the value of $T_\lambda(0.005)$ to zero concentration of He^3. The determination of $T_\lambda(x_3)$ and $T_\lambda(0)$ was described in the section EXPERIMENTAL PROCEDURE AND APPARATUS.

For the mixtures, the values of $(\eta\rho_n)$ at the lambda point, $(\eta\rho_n)_\lambda$, were determined from smoothed expanded plots of the curves in figure 4 by noting the ordinates corresponding to the values of $T_\lambda(x_3)$. It was decided that, for pure He^4, a better estimate of $(\eta\rho_n)_\lambda$ could be obtained by plotting $(\eta\rho_n)_\lambda$'s for the mixtures against x_3 and extrapolating to zero He^3 concentration.

These procedures yield only the experimental values of $(\eta\rho_n)_\lambda$ and $T_\lambda(x_3)$. However, these quantities are the ones which are relevant for comparing results among different concentrations, since the "true" absolute values were not determined.

The absolute values of $\eta\rho_n$ in the present experiment appear to be about 1.4×10^{-5} (N-sec/m^2) (kg/m^3) (0.14 micropoise g cm^{-3}) too high (about 4 percent too high at T_λ for pure He^4) compared to the values reported in references 5 and 6, which are probably more reliable. (A comparison at lower temperatures leads us to believe that the crystal was slightly cocked in the present experiment.) With regard to the temperature, we conclude that approximate values of temperature on the T_{58} scale can be obtained by subtracting 1.7 millikelvins from the experimental temperatures on the abscissas of figures 4 and 6.

It has also been possible to use the present results to correct the absolute temperature on the T_{58} scale shown in table I of references 5 and 6. The correction is based on a finding that the pure He^4 data from these earlier measurements can best be made consistent with the present data if the lambda point in these references is moved from the high-temperature side of the data gap to the low-temperature side. In these references the location of the lambda point of pure He^4 was somewhat uncertain due to the absence of data over a 2-millikelvin interval which included $T_\lambda(0)$. The lambda point was taken to be near the high-temperature side of the data gap partly because this side of the data gap corresponded to a measured experimental temperature of 2.172 K. The present investigation has demonstrated that none of our experimental temperatures could be used to determine the lambda point of pure He^4 more accurately than within a few millikelvins. Thus, the aforementioned lambda point in references 5 and 6 could

actually have been anywhere within the data gap, and some other factors would have to be considered in deciding where the lambda point lay.

A doubly reduced plot of the pure He^4 data from reference 5 was made and compared with the data from the present investigation. It was found that the two plots could be made nearly identical if the lambda point in reference 5 were moved from the higher temperature side of the data gap to the lower temperature side. We have, therefore, concluded that the results of reference 5 should be corrected. This correction requires that 1.4 millikelvins must be added to all temperatures listed in table I of references 5 and 6. With this correction, the temperature values listed in the table are the absolute temperatures on the T_{58} scale with $T_\lambda(0)$ at 2.172 K.

The viscosity of the mixtures was also of interest. This quantity was derived from the $\eta\rho_n$ values by using the calculated values of ρ_n. The variation of ρ_n with T is so rapid near $T_\lambda(x_3)$ that serious errors in η could result from small errors in the

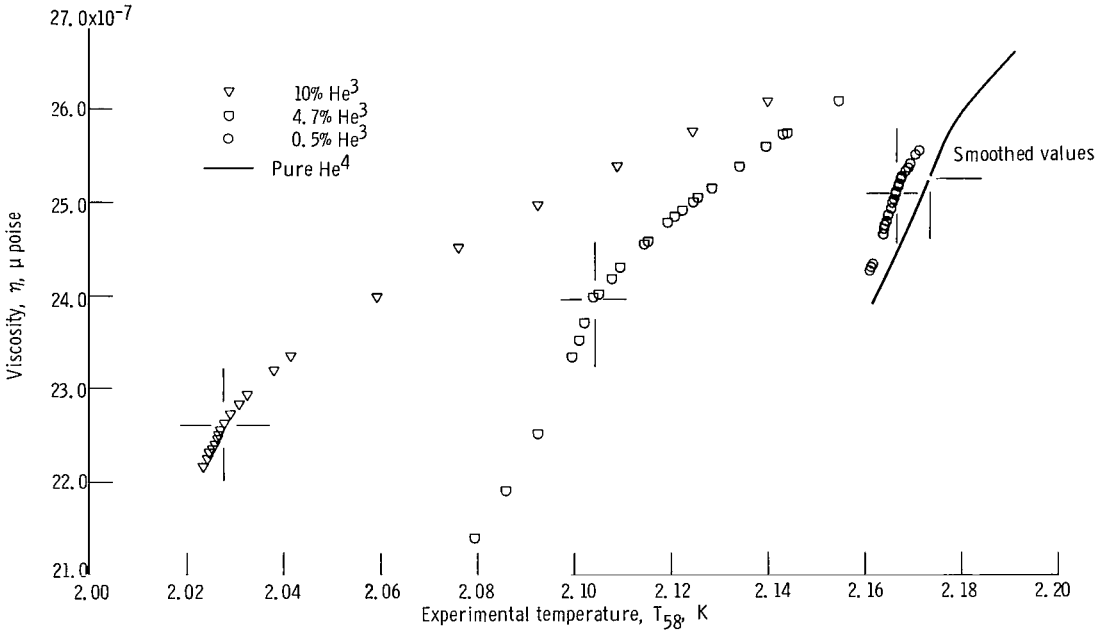


Figure 6. - Viscosity as a function of experimental temperature for pure He^4 and dilute He^3-He^4 mixtures.

absolute temperature. Therefore, ρ_n was computed as a function of T^*, which we know much better than T absolute. Figure 6 shows η against $T(x_3)$ for pure He^4 and for three mixtures. The results show that, as was mentioned earlier, η decreases with x_3 at a given value of $T - T_\lambda$.

This conclusion is not meant to be extrapolated, for $T(x_3) < T_\lambda(x_3)$, to values of $T(x_3) - T_\lambda(x_3)$ larger than those shown in figure 6. In addition, it should be kept in mind

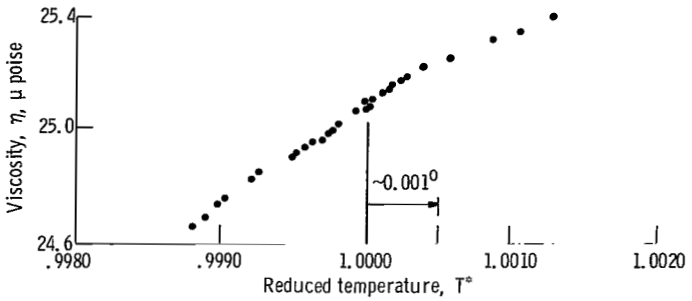


Figure 7. - Viscosity as function of reduced temperature for 0.5-percent-He^3 mixture.

that the computation of $\rho_n(T^*, x_3)$ cannot be reliably extrapolated to values of T^* and x_3 outside of the range used here.

Figure 7 is an expanded plot of η against T^* for an x_3 of 0.005. This curve shows that neither η nor $d\eta/dT$ is discontinuous across T_λ. The conclusion to be drawn from this figure is that the sharp change in $\eta\rho_n$ across T_λ are due entirely to ρ_n. Figure 8 shows, in a less precise manner, that this is also true for the 0.1005-mole-fraction mixture. The significance of the apparent slight change in slope of η against T^* at a temperature about 1 millikelvin above the lambda point is not clear. However, as can be seen from figure 5(b), it can be stated with confidence that this

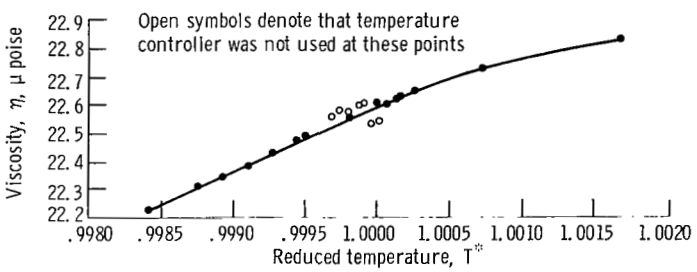


Figure 8. - Viscosity as function of reduced temperature for 10-percent-He^3 mixture.

break in the slope of the η-against-T^* curve cannot be due to an improperly determined lambda point.

Another point worth noting is shown in figure 9. Let $\eta_\lambda(x_3)$ denote the viscosity at $T_\lambda(x_3)$ for a He^3 concentration given by x_3. Then figure 9 shows that the dependence of $\eta_\lambda(x_3)$ on $T_\lambda(x_3)$ is well fitted by a straight line. This result does not seem to have been published heretofore. It has been mentioned earlier that $T_\lambda(x_3)$ is closely approximated by $(1 - x_3)^{2/3}T_\lambda(0)$ (refs. 10 and 11). Therefore, it may be concluded from figure 9 that

$$\eta_\lambda(x_3) - \eta_\lambda(0) = A\left(1 - x_4^{2/3}\right)T_\lambda(0)$$

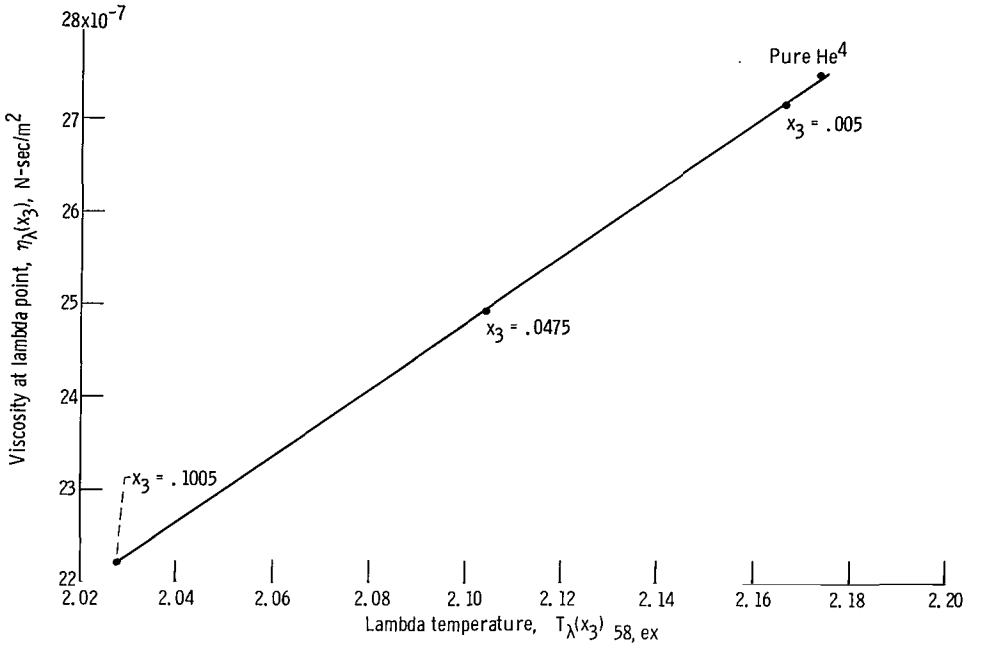


Figure 9. - Viscosity at lambda point as function of lambda temperature for pure He^4 and three He^3-He^4 mixtures. (The straight line was drawn through points corresponding to $x_3 = 0.005$ and $x_3 = 0.1005$ (the most reliable points). For pure He^4, $T_\lambda(0)$ was obtained by adding a correction to $T_\lambda(0.005)$; $\eta_\lambda(0)$ was obtained as follows: First, a plot of $\eta\rho_n$ against x_3 was extrapolated to zero He^3 concentration; then, this value was divided by the accepted value of ρ_n at $T_\lambda(0)$, which is 146.2 kg/m^3.

where A is a constant equal to the negative of the slope of the line in figure 9. This result does not seem to have been published heretofore.

Finally, we wish to set forth our reasons for believing that η and $d\eta/dT$ of pure He^4 are continuous across $T_\lambda(0)$. Of course, the obvious points that η and $d\eta/dT$ are very smooth across T_λ for the 0.5-percent mixture and that 0.5 percent is only a small amount of He^3 are important parts of the argument. However, the case is somewhat stronger than this. First of all, the value of $T_\lambda(0)$ was obtained by adjusting $T_\lambda(x_3)$ to zero concentration of He^3, and $(\eta\rho_n)_\lambda$ was obtained by extrapolating the $\eta\rho_n$ curve for the mixtures, as explained previously. The doubly reduced plots of $\eta\rho_n$ against T for pure He^4 using these values blend smoothly into the curves for the mixtures (taking into consideration the comparatively poor precision of the He^4 data near $T_\lambda(0)$). The same behavior is evident for the 10-percent solution, as shown in figure 8. Secondly, there does not seem to be any tendency for η to become ill behaved as x_3 decreases. The normal density, of course does have a discontinuous slope at $T_\lambda(x_3)$.

We feel that, taken together, these facts indicate strongly that η and $d\eta/dT$ are both likely to be continuous across $T_\lambda(0)$.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, July 21, 1971,
112-02.

REFERENCES

1. Reynolds, Joseph M.; Hussey, Robert G.; Thibodeaux, Devron P.; Tucker, Bert E.; and Urrechaga-Altuna, Jesus: The Oscillation of Cylinders and Spheres in Liquid Helium II. Louisiana State Univ. (AFASD-TDR-63-817, pt. 1, DDC No. AD-428267), Dec. 1963.

2. Welber, Benjamin: Damping of a Torsionally Oscillating Cylinder in Liquid Helium at Various Temperatures and Densities. Phys. Rev., vol. 119, no. 6, Sept. 15, 1960, pp. 1816-1822.

3. Dash, J. G.; and Taylor, R. Dean: Density and Viscosity of Normal Fluid in Dilute Solutions of He^3 in He^4. Phys. Rev., vol. 107, no. 5, Sept. 1, 1957, pp. 1228-1237.

4. Staas, F. A.; Taconis, K. W.; and Fokkens, K.: Viscosity of Liquid ^3He-^4He Mixtures in the Helium II Region Above 1° K. Physica, vol. 26, 1960, pp. 669-686.

5. Webeler, Rayjor W. H.; and Hammer, David C.: Damping of a Torsional Oscillator in Liquid Helium 4 and 3 From 0.4° to 2.5° K. NASA TN D-4381, 1968.

6. Webeler, R. W. H.; and Hammer, D. C.: Viscosity × Normal Density of Liquid Helium in a Temperature Interval about the Lambda Point. Phys. Letters, vol. 15, no. 3, Apr. 1, 1965, pp. 233-234.

7. Welber, Benjamin; and Quimby, S. L.: Measurement of the Product of Viscosity and Density of Liquid Helium with a Torsional Crystal. Phys. Rev., vol. 107, no. 3, Aug. 1, 1957, pp. 645-646.

8. Kerr, Eugene C.; and Taylor, R. Dean: Molar Volume and Expansion Coefficient of Liquid He^3. Ann. Phys. (N. Y.), vol. 20, no. 3, Dec. 1962, pp. 450-463.

9. Kerr, Eugene C.; and Taylor, R. Dean: The Molar Volume and Expansion Coefficient of Liquid He^4. Ann. Phys. (N. Y.), vol. 26, no. 2, Feb. 5, 1964, pp. 292-306.

10. Clow, James R.; and Reppy, John D.: Temperature Dependence of the Superfluid Density in He II near T_λ. Phys. Rev. Letters, vol. 16, no. 20, May 16, 1966, pp. 887-889.

11. Tyson, J. A.; and Douglass, D. H., Jr.: Superfluid Density and Scaling Laws for Liquid Helium near T_λ. Phys. Rev. Letters, vol. 17, no. 9, Aug. 29, 1966, pp. 472-474.

NASA-Langley, 1971 —— 23 E-6448

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546